Microwave Measurement of a Complex Dielectric Constant Over a Wide Range of Values by Means of a Waveguide-Resonator Method

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Abstract—A new method of measuring complex dielectric constants at microwave frequencies by introducing a resonant circuit comprised of the experimental sample within a waveguide is described. The theoretical evaluation of working equations for the complex dielectric constant under the quasi-stationary distribution of the field within the sample is given. In this case, the equations are derived 1) considering the elements as lumped parameters and 2) distributed parameters are treated by means of transmission line equations. The two sets of equations are compared.

The working equations are also derived for the condition when the experimental sample takes up the form of a radial line. The accuracy of determining the parameters is computed and experimental results are provided as verification of the applicability of the given method.

INTRODUCTION

In the given work, a new method of measurement of complex dielectric constant over a wide range of values is suggested. The basic principle of the method is a combination of waveguide and the resonator method.

The resonant unit consists of a tunable coaxial line, the central conductor of which is extended as a probe within the waveguide. The experimental sample is placed between the end of the probe and the wall of the waveguide as shown in Fig. 1.

In order to increase the effect of the experimental sample on the measurable parameters, the height of the waveguide is reduced and this is subsequently matched to the standard waveguide by means of a Chebyshev impedance matching transformer.

During the quasi-stationary distribution of fields within the experimental sample, resonance is obtained by means of the tunable coaxial line and the real part of the complex dielectric constant can be expressed as a function of the resonance value of the coaxial line reactance.

Tan δ of the experimental sample at quasi-stationary distribution of field is determined as a function of the reflection coefficient at resonance and also from the change of Q of the resonant contour after the placement of the experimental sample.

The working equations for ϵ' and tan δ of the experimental samples under the quasi-stationary distribution of field are derived considering the resonator elements as lumped parameters. More exact equations are derived with the help of transmission line equations under the specific boundary conditions. Convergence of the two sets of equations is evaluated for the specific values of the resonant elements.

With the increasing ϵ' or frequency, the condition for the quasi-stationary distribution of field within the sample fails to satisfy and subsequently the disk sample takes up the form of a radial line having ordered resonant and antiresonant frequencies. In such cases, the input impedance of the experimental sample will have successive impedance maxima and minima. By noting the frequency difference between the two successive antiresonant points, ϵ' of the experimental sample can be determined.

However, resonance frequencies of the experimental sample could also be used to determine ϵ', but it would have less effect on the measurable parameters and consequently the accuracy of the measurement would be less than the corresponding antiresonant measurements. The real difficulty encountered during the measurement of the complex dielectric constant of the experimental sample in the form of a radial line is in the exact elimination of reactance of other elements in the resonant contour.

Tan δ of the experimental sample can be determined from the magnitude of the reflection coefficient at one of the lower antiresonant points of the sample.

At further increment of ϵ' of the experimental sample, as in the case of certain ferro-electric materials near the point of phase transition, the input impedance of the experimental sample is quite low in order to have appreciable effect on the measurable parameters and consequently, the accuracy of determining ϵ', tan δ goes down. In order to increase the input impedance of the experimental sample, a $\Lambda/4$ impedance transformer can be used as shown in Fig. 6.

Accuracy of determining ϵ', tan δ in these cases is evaluated and the experimental results are provided verifying the applicability of the method at different ranges of ϵ', tan δ values. The possibility of thermal shielding of the experimental sample in this method makes it easier to study the thermal behavior of dielectric properties.

THEORY

Quasi-Stationary Distribution of Field within the Sample

Case 1—Lumped Parameter Concept: When the condition of quasi-stationary distribution of field within the experimental sample is satisfied, the system can be represented

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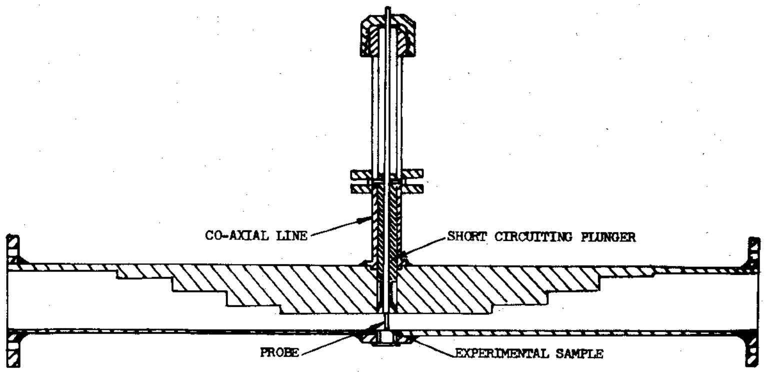

Fig. 1. Experimental unit.

by means of an equivalent circuit as shown in Fig. 2, where

- X_K reactance of the tunable coaxial line;
- X_C capacitive reactance of the experimental sample;
- X_L reactance of the probe;
- R_C $X_C \tan \delta$. Equivalent resistance due to losses in the experimental sample;
- R_T equivalent losses in the measurement system, i.e., wall of the waveguide, probe, and tunable coaxial line.

If the VSWR of the system without probe is equal to unity

$$Z_0 \tan kl_1 + X_L = 0 \quad (1)$$

where Z_0 is the characteristic impedance of the coaxial line.

l_1 is the length of the short-circuited coaxial line, tuned to resonance with probe only. If l_2 is the corresponding length of the coaxial line for resonance with the experimental sample

$$Z_0 \tan kl_2 - X_c + X_L = 0. \quad (2)$$

Substituting (1) in (2), the reactance of the experimental sample can be expressed as

$$X_C = \left[\frac{Z_0 \sin k(l_2 - l_1)}{\cos kl_1 \cos kl_2}\right]$$

$$X_C = \frac{1}{2\pi fC} \qquad C = \frac{\epsilon' \epsilon_0 \pi r^2}{d} \quad (3)$$

where

- ϵ_0 permittivity of free space;
- r radius of the experimental sample in meters;
- d thickness of the sample in meters.

Tan δ of the sample can be determined with the help of the reflection coefficient of the system tuned to resonance. If the losses in the waveguide system are not high, assuring VSWR < 1.1, the losses in the system without the experimental sample can be determined with the help of the reflection coefficient of the system tuned to resonance

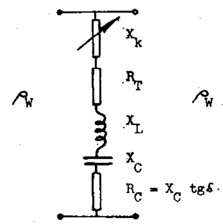

Fig. 2. Equivalent circuit of the experimental unit.

without the experimental sample. Tan δ of the sample, as a function of these reflection coefficients can be expressed by means of the following equation:

$$\tan \delta = \frac{2\rho_w}{x_c}\left[\frac{1}{1 - |\Gamma_T|} - \frac{1}{2} - \frac{1 + |\Gamma_L|}{2[1 - |\Gamma_L|]}\right] \quad (4)$$

where

- ρ_w characteristic impedance of the reduced height waveguide at the position of the experimental sample;
- Γ_T reflection coefficient of the system with the sample tuned to resonance;
- Γ_L reflection coefficient of the system without the sample tuned to resonance.

Tan δ of the sample can also be determined from the change of Q of the resonant system after the placement of the sample. If l_2 as denoted earlier is less than $\Lambda/4$, i.e., for the samples with low ϵ', the tan δ of the sample can be determined with the help of the equation

$$\tan \delta \cong \frac{1}{Q'} + \frac{\sin kl_1 \cos kl_2}{Q_0 \sin k(l_2 - l_1)} \quad (5)$$

where Q' is the loaded Q of the resonant system and Q_0 is the unloaded Q of the resonant contour.

If $l_2 > \Lambda/4$, i.e., for the samples with high ϵ', tan δ can be expressed by the following equation:

$$\tan \delta \cong \left[\frac{1}{Q_0} - \frac{1}{Q'}\right] \frac{\sin kl_1 \cos kl_2}{\sin k(l_2 - L_1)}. \quad (6)$$

For lossy samples with low ϵ', it is difficult to determine the loaded Q because, with detuning, the reactance of the coaxial line and that of the probe change. The effect of this detuning has been taken into account in the work [1].

Quasi-Stationary Distribution of Field within the Sample

Case 1—Derivation of Working Equations with the Help of Transmission Line Equations: In the given case, the equation for the influence of the experimental sample on the reflection coefficient is derived with the help of transmission line equations. It can be assumed that the distribution of current along the probe can be quite accurately represented by the following equations:

$$\frac{d\phi}{dy} = ZI(y) + E_R \tag{7}$$

$$\frac{dI}{dy} = -Y\phi \tag{8}$$

where

Y distributed admittance of the probe;
Z distributed impedance of the probe;
ϕ scalar potential along the length of the probe;
E_R resultant electric field along the surface of the probe.

$$E_R = E^+ + E^-.$$

Assuming that the distribution of current along the probe is known, the amplitude of the reflected wave can be derived with the help of the equation

$$E^- = \frac{1}{N_S}\int I(y)E^+\,dy \tag{9}$$

where

E^+ amplitude of the incident wave;
$I(y)$ function of current distribution along the probe;
N_S norma.

For a waveguide excited by the TE_{01} mode of operation, N_S can be given by [4]

$$N_S = \frac{-ab}{120\pi}(E^+)^2\left[1-\left(\frac{\Lambda}{2a}\right)^2\right]^{1/2} \tag{10}$$

where a is the width of the waveguide and b is the height of the waveguide. It is assumed that the characteristic impedance of a probe can be approximately represented with the following equation [2]:

$$\rho_P = 60\left\{2.303\log\frac{\Lambda}{2\pi r_p} + 0.116\right\} \tag{11}$$

where r_p is the radius of the probe.

Applications of the above equation which is derived from antenna theory [2] in the unbounded medium, is the basic assumption made for the given problem. Subsequent experimental verification of the characteristic impedance of the probe with an accuracy better than 4 percent under given experimental limitations, justifies the previous assumptions regarding current distribution along the probe, so also the applicability of transmission line equations within the bounded medium with certain approximations.

If the solutions of (7) and (8) are represented by the following equations:

$$\phi = A\sin ky + B\cos ky \tag{12}$$

$$I = C\sin ky + D\cos ky - j\frac{E^+}{k\rho_P} \tag{13}$$

the constants A, B, C, and D can be determined with the help of the boundary conditions:

$$\phi\,|_{y=o} = 0 \tag{14a}$$

$$\phi\,|_{y=b-o} = jI_y\,|_{y=b-o} \times x_{\text{coax}} \tag{14b}$$

$$\phi\,|_{y=o+o} = I_y\,|_{y=o+o} \times x_c \tag{14c}$$

where x_{coax} is the reactance of the coaxial line and x_c is the reactance of the sample.

After determination of the constants C and D with the help of boundary conditions, it is possible to get an expression for the reflection coefficient by means of (7), (9), and (12).

In the case of experimental samples without losses, the equation for reflection coefficient can be expressed by

$$\Gamma = \frac{j\eta}{\mu - j\eta} \tag{15}$$

where μ and η are functions of N_S, ρ_P, X_C, X_K and can be represented by

$$\eta = \frac{j}{k\rho_P N_S}\left\{\frac{1}{\rho_P\sin kb} - \frac{\cot kb}{\rho_P}\right\} \tag{16}$$

$$\mu = \left\{1 + \frac{X_C X_K}{\rho_P^2} + \frac{\cot kb}{\rho_P}(X_K - X_C)\right\}. \tag{17}$$

At resonance $d\Gamma/dX_K = 0$ and correspondingly the reactance of the coaxial line at resonance can be expressed as a function of reactance of the experimental sample by means of the following equation:

$$X_K = \rho_P\frac{[X_C\cot kb - \rho_P]}{[X_C + \rho_P\cot kb]}. \tag{18}$$

The expression for reflection coefficient from a short-circuited probe takes up the form

$$\Gamma = \frac{-b/k\rho_P N_S}{1 + j(b/k\rho_P N_S)}. \tag{19}$$

A similar problem has been solved [3] for a waveguide shunted by a probe and the equation for the reflection coefficient in this case is given by

$$\Gamma = \frac{-1}{1 + 2jX}. \tag{20}$$

The magnitude of the reactance as put by the probe across the waveguide can be evaluated with the help of the following equation [3]:

$$X = \frac{a}{2\Lambda_g} \csc\frac{\pi d}{a}\left[\ln\frac{2a}{\pi r_p}\cdot\sin\frac{\pi d}{a} - \sin^2\frac{\pi d}{a}\left(2+\frac{k^2a^2}{\pi^2}\right) + k^2d^2\left(-\ln\frac{2\pi d}{a}+\frac{3}{2}+\frac{\pi^2d^2}{36a^2}\right)\right]. \tag{21}$$

where

- r_p radius of the probe;
- d distance of the center of the probe from the wall of the waveguide (in the given case $d = a/2$);
- Λ_g guide wavelength;
- a width of the waveguide.

The reflection coefficients for identical configurations as calculated for $r_p = 0.75$ mm and $\Lambda_0 = 10$ cm for the S-band waveguide according to (19) and (20) are, respectively, $(-0{,}17 + j0.432)$ and $(-0{,}164 + j0.31)$. By replacing $X_C = X_C\ (1 - j\tan\delta)$ in (14c), $\tan\delta$ of the experimental sample can be determined with the help of the following equation:

$$|\Gamma| = \frac{-\{k\beta X_C\tan\delta - K\xi bX_C\tan\delta\}^2 + K\zeta\{\xi X_C\tan\delta - K\zeta\}}{\{K\beta X_C\tan\delta - K\xi bX_C\tan\delta\}^2 + \{\xi X_C\tan\delta - \xi K\}^2} \tag{22}$$

where

$$K = \frac{1}{k\rho_P N_S} \qquad \xi = \left[\frac{X_K}{\rho_P^2} - \frac{\cot kb}{\rho_P}\right]$$

$$\beta = \left\{\frac{2X_K}{k\rho_P^2}\left(\frac{1}{\sin kb} - \cot kb\right) - \frac{1}{k\rho_P}\right\}$$

$$\zeta = \left[\frac{2X_CX_K}{k\rho_P^2}\left\{\frac{1}{\sin kb} - \cot kb\right\} + \{X_K - X_C\}\right].$$

$|\Gamma|$ is the reflection coefficient at resonance. For smaller values of $\tan\delta$, (22) can be simplified in the following form:

$$\tan\delta \cong \frac{K\zeta\{|\Gamma| - 1\}}{\xi|\Gamma|}. \tag{23}$$

The effect of losses in the system can be taken into account by assigning a complex value to the characteristic impedance of the probe. However, it is more complicated to determine the value of $\tan\delta$ by taking into account the losses in the system. For this purpose, (4) can be used more conveniently than the previous method. For systems with extremely low losses and for moderate values of $10^{-1} < \tan\delta < 10^{-2}$, (23) gives more satisfactory results. Equation (18) as derived with the help of transmission line equations is more exact than (3) for evaluating the values of ϵ', as it was subsequently verified experimentally.

Resonance values of coaxial line reactances for different values of ϵ' are calculated with the help of (3) and (18). It can be seen from Fig. 3 that the two equations converge for $X_K = 0$, i.e., when the capacitive reactance of the experimental samples is tuned only by the inductive reactance of the probe; and from the accuracy curve [Fig. 4(a) and (b)] it can be seen that the ϵ' of the experimental sample in this region can be determined with the best accuracy. Subsequent experimental verification shows that ϵ' of the experimental sample can be evaluated more accurately with the help of (18).

Accuracy: From the accuracy curves for determining Fig. 4(a) and (b), it can be seen that the ϵ' can be determined with satisfactory accuracy (in the range of 30–120) by proper adjustment of sample dimensions, (i.e., thickness and diameter) and the characteristic impedance of the probe (i.e., by changing the diameter of the probe).

Increase in error in the determination of ϵ' for lower and higher values of ϵ' can be explained by the fact that lower values of ϵ', for which the coaxial line is tuned more towards $\Lambda/4$, produce large error due to the inaccuracy of determination of the length of the coaxial line.

For higher values of ϵ', inaccuracy grows due to the errors in the determination of sample dimensions. However, the effect of fringing field has been neglected in all these calculations. Any misalignment of the experimental sample with the probe will enhance the effect of fringing field and an attempt has been made to minimize any error due to this by extremely careful alignment of the experimental sample with the probe, and very good contact of the sample with the waveguide by silvering the sample. In the present geometry of the experimental setup the fringing field effect can be neglected in the first-order approximation. The given curves estimate the accuracy mainly due to the error in measurements of sample dimensions and frequency instability of the generator. The effect of fringing field which is certain to influence the accuracy of measurement has not been taken into account. The samples are selected for which the condition of quasi-stationary distribution of field is well satisfied (i.e., $d/(\epsilon')^{1/2} \ll \Lambda/2$ and $r/(\epsilon')^{1/2} \ll \Lambda/2$).

But the limiting cases for which the sample does not take up the form of radial line, and for which, at the same time, the condition of quasi-stationary distribution of field is not well satisfied, are not studied in the present work. In such cases, the anomally can be partly reduced by employing samples of special shapes, which is a separate subject of detailed study.

The accuracy curves for $\tan\delta$ [Fig. 5(a) and (b)], resemble those of resonator methods where errors grow with

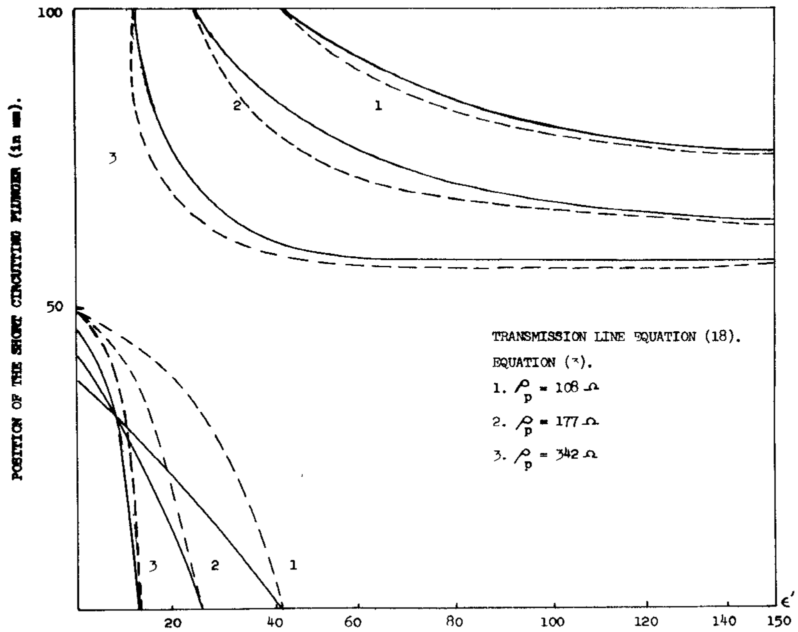

Fig. 3. Comparative values of coaxial resonant length according to (3) and (18).

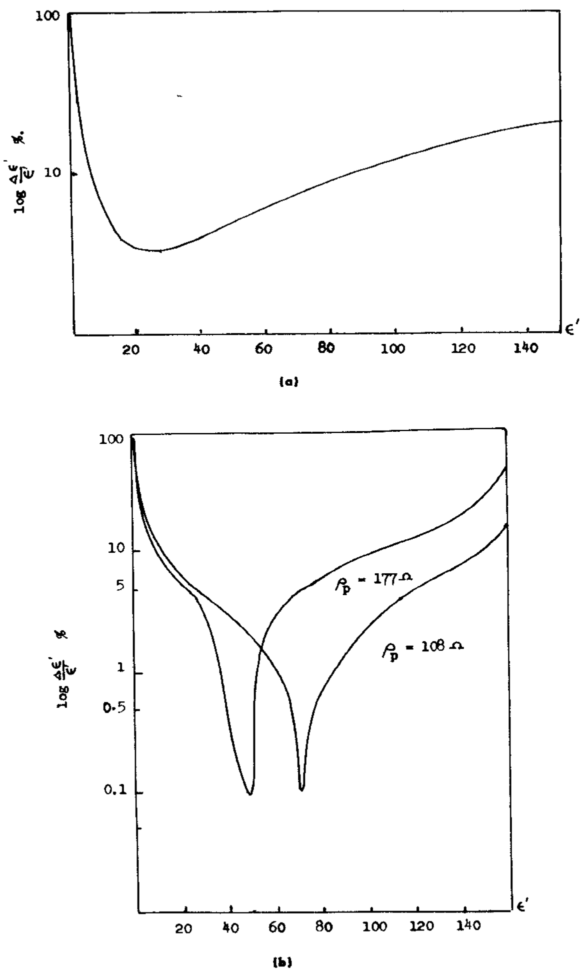

Fig. 4. (a) Accuracy curve of determination of ϵ' by (3). (b) Accuracy curve of determination of ϵ' by (18).

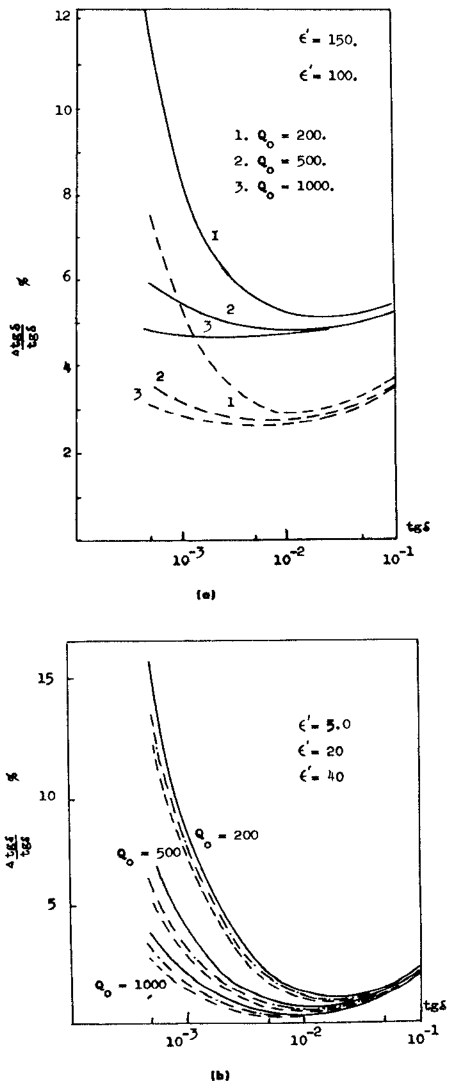

Fig. 5. (a) Accuracy of determination of tan δ. (b) Accuracy of determination of tan δ.

reducing values of tan δ, as the losses in the experimental sample are comparable with the losses in the system and for higher values of tan δ, the resonance curves become quite flat, so that the error increases in the measurement of the loaded Q.

For very low lossy samples, the loaded Q of the resonant contour increases due to redistribution of current in it. This is observed experimentally, in the case of a low-loss ceramic (Fig. 8) where the resonance curve shows more reflection than in the case of resonance with a short-circuited probe.

Case 2—The Sample in the Form of a Radial Line: With increasing values of ϵ' and that of frequency, it becomes difficult to satisfy the condition of quasi-stationary distribution of field within the experimental sample and by further increasing ϵ' (or frequency), the disk sample takes the form of a radial line. If the thickness of the sample is much less than $\Lambda/2$, field distribution within this radial line will be of the TM_{00} type [4] and the field components within it can be represented by

$$E_z = E_0 J_0(kr) \tag{24}$$

where

$$H_\theta = -\frac{j\omega\epsilon'}{k}\frac{\partial E_z}{\partial r} = j\omega\epsilon' E_0 J_1(kr) \tag{25}$$

$$k = \frac{2\pi}{\Lambda} = \omega(\mu\epsilon)^{1/2} = \omega(\mu\epsilon_0)^{1/2}(\epsilon' - j\epsilon'')^{1/2}$$

$J_0(x)$ is the Bessel function of zero order and $J_1(x)$ is the Bessel function of first order.

The coupling of field of the waveguide and that of the sample takes place through the continuity of electric-field components, whereas in the subsequent modified version with the $\Lambda/4$ coaxial line transformer, coupling takes place through the continuity of magnetic field components.

The equation for input impedance of such a radial line can be derived from

$$Z = -\frac{jd}{2\pi R}\cdot\frac{k}{\omega\epsilon}\frac{J_0(kr)}{J_1(kr)} \tag{26}$$

where d is the thickness of the sample and R is the radius of the sample.

By expanding (26) with the help of Taylor's series and neglecting the second-order terms, the expression for input impedance takes the form

$$Z \cong -\frac{jd\cdot 120\pi}{2\pi R(\epsilon')^{1/2}}\left[\frac{J_0(x) - jx\tan\delta J_0'(x)}{J_1(x) - jx\tan\delta J_1'(x)}\right] \tag{27}$$

where $J_0'(x)$ and $J_1'(x)$ are derivatives of $J_0(x)$ and $J_1(x)$, respectively.

By using recurrence relations of the Bessel functions, the final expression for input impedance for such a radial line can be expressed as

$$Z \cong -\frac{j120d}{R(\epsilon')^{1/2}}\left\{\frac{[8J_0J_1 - x^2\tan^2\delta J_1(J_0 - J_1)] + jx\tan\delta[4J_1^2 + 2J_0(J_0 - J_2)]}{16J_1^2 + x^2\tan^2\delta(J_0 - J_2)^2}\right\}. \tag{28}$$

The input impedance of such a radial has got repetitive resonant and antiresonant frequencies. It can be shown from (28) that the difference between two successive resonant or antiresonant roots of Bessel function is equal to π. Therefore, by measuring the difference in frequencies of two successive antiresonant points, ϵ' of the sample can be determined from the equation

$$x_1 - x_2 = \pi = [\omega_1 - \omega_2](\mu\epsilon')^{1/2}R$$

$$(\epsilon')^{1/2} = \frac{C_0}{2}[f_1 - f_2]R. \tag{29}$$

The input impedance of the experimental sample at the antiresonant (AR) point takes the form

$$Z_{AR} \cong \left\{\frac{120d}{R(\epsilon')^{1/2}X_{AR}\cdot\tan\delta}\right\} \tag{30}$$

where $X_{AR} = 3.83 + n\pi$, $n = 0, 1, 2, 3\cdots$, and the resonance value of the input impedance of the experimental sample

$$Z_R \simeq \frac{120d}{R(\epsilon')^{1/2}}\frac{X_R\cdot\tan\delta}{4[1 + \tan^2\delta]} \tag{31}$$

$$X_R \simeq 3.83 + (n + \tfrac{1}{2})\pi, \qquad n = -1, 0, 1, 2, 3,\cdots.$$

In measurements it is more effective to note the antiresonant points, because at resonance, the input impedance of the sample (which is mainly due to the loss term tan δ) is likely to be comparable with the losses in the system and consequently, the accuracy of measurement will be less.

Tan δ of the experimental sample can be determined with the help of the reflection coefficient of the system tuned to antiresonance of the experimental sample. The reactance term of the probe at this frequency should be cancelled out by means of the tunable coaxial line, from the predetermined values for the corresponding frequency. Losses in the system can be approximately taken into account with the help of the reflection coefficient from the system tuned to resonance with the short-circuited probe.

The equation for tan δ can be then expressed as

$$\tan\delta \cong \frac{120d}{[(1 - \Gamma_{AR})/2\Gamma_{AR} + (1 - \Gamma_L)/(1 + \Gamma_L)]Z_{0_w}\cdot R(\epsilon')^{1/2}X_{AR}} \tag{32}$$

where

Z_{0_w} characteristic impedance of the waveguide at the position of the placement of the sample;

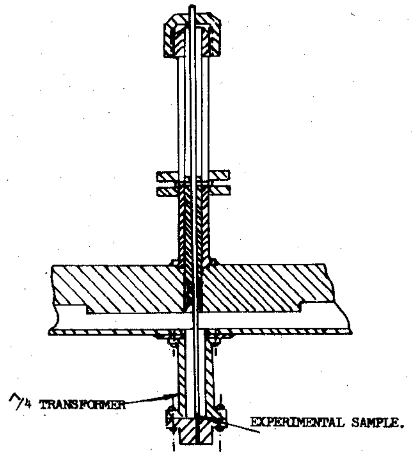

Fig. 6. Modified unit to measure high ϵ'.

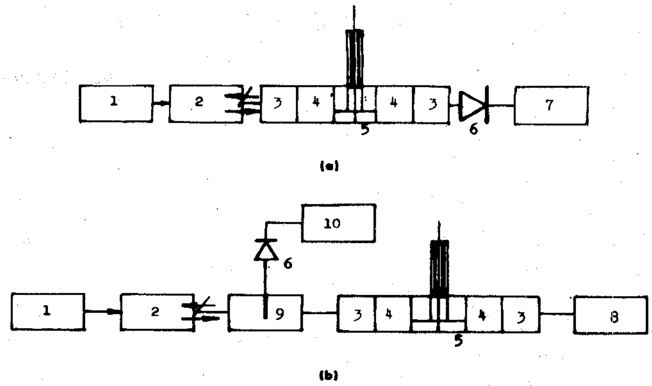

Fig. 7. 1—generator; 2—isolator; 3—waveguide coaxial transformer; 4—Chebyshev impedance transformer; 5—experimental unit; 6—detector; 7—power meter; 8—matched load; 9—slotted line; 10—VSWR meter. (a) Schematic representation of setup to plot resonance curve. (b) Schematic representation of setup to measure reflection coefficient at resonance.

Γ_{AR} reflection coefficient from the system with the sample at antiresonance;

Γ_L reflection coefficient from the system with the short-circuited probe tuned to resonance.

In order to reduce error in the measurement of $\tan\delta$, the reflection coefficient VS frequency should be measured by properly cancelling out the reactance due to the probe. Any residual reactance of the probe is likely to show not only the increased value of $\tan\delta$ but also some shift of antiresonant frequency of the sample and a corresponding error in the measurement of ϵ'.

ϵ' can be determined with an accuracy better than 2 percent and accuracy of determining $\tan\delta$ is higher for lower values of $\tan\delta$ and reduces with increasing values of $\tan\delta$. This trend is opposite in the case of samples with quasi-stationary distribution of field in it. Accuracy in measurement of $\tan\delta$ is within 10–12 percent for $\tan\delta$ 10^{-3}–10^{-2} and within 12–30 percent for $\tan\delta$ 10^{-2}–10^{-1}.

For very high values of $\epsilon'(\epsilon' > 5000)$, as in the case of a few ferro-electrics near the temperature of phase transition, the input impedance is quite low in order to have significant effect on the measurable parameter. The input impedance of such samples can be stepped up with the help of a $\Lambda/4$ impedance transformer as shown in Fig. 6.

Experimental Results

The schematic experimental setup is shown in Fig. 7(a) and (b). The setup as shown in Fig. 8(a) is used to determine quickly the resonant point for determination of ϵ'. Keeping the system tuned to resonance, the setup as shown in Fig. 7(b) is used to determine the reflection coefficient, at this point, in order to find out the value of $\tan\delta$. Typical experimental resonance curves are shown in Fig. 8 for the short-circuited probe and two ceramics.

It is seen that the characteristic impedance of the probe is determined with an accuracy better than 4 percent. In order to verify the applicability of the method in the case of resonant samples VK 7, a specially synthesized material from $BaTiO_3$ and $SnTiO_3$ was used as the experimental sample.

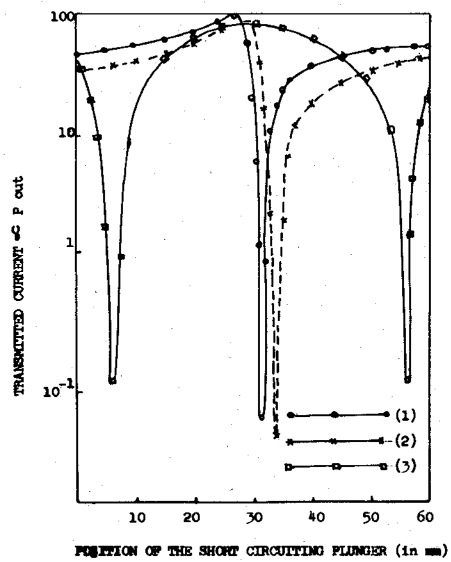

Fig. 8. Experimental resonance curves. (1)—short-circuited probe; (2)—rutile, ϵ' = 155; (3)—ceramic, ϵ' = 20.

ϵ' and $\tan\delta$ of this sample as measured by coaxial line by noting VSWR and minimum position were, respectively, 2620 and 1.35×10^{-3} at room temperature. ϵ' and $\tan\delta$ of this sample as measured by the suggested method were, respectively, 2600 and 1.5×10^{-3} at room temperature. Experimental results for a few samples with the quasi-stationary distribution of field within the samples are tabulated in Table I.

TABLE I

MATERIAL	BY THIS METHOD		BY OTHER METHOD		OTHER METHOD
	ϵ'	$tg\delta$	ϵ'	$tg\delta$	
Viniplast	3.43 - 4.0	(5-6) x 10^{-2}	3.8 - 4.0	5 x 10^{-2}	[5]
Transparent Plastic	3.31 - 3.6	(4.5 - 6) x 10^{-2}	3.0 - 3.3	5.5 x 10^{-2}	[5]
Sital	9.3 - 9.5	4.1 x 10^{-3}	8.8 - 9.0	3.0 x 10^{-3}	[5]
Ceramic	18.5 - 20	5.5 x 10^{-3}	20	4.0 x 10^{-3}	[6]
Rutile	162	-	156	(7 - 9) x 10^{-3}	[6]
Probe	Characteristic impedance 183 Ω		Theoretical value - 190 Ω		

CONCLUSION

The proposed method which is based on the combined principle of the resonator and waveguide method can be used with certain modification over a wide range of values of ϵ' and tan δ. This method can be considered to be universal like waveguide or coaxial line methods [6]–[10].

However, the waveguide and coaxial line methods which can be used to measure ϵ' and tan δ over a wide range of values are to employ graphical solution in certain cases in order to solve nonsingle valued transcendental equations [7]. Simpler working equations in closed form in the given case can be considered to be an added advantage of this method over the earlier methods, which enables measurement of ϵ' and tan δ over wide range of values.

In certain cases of the earlier methods, fabrication of the experimental sample, in order to satisfy the condition of quasi-stationary distribution of field, requires a very sophisticated technological setup [9]. The same experimental unit cannot always be used for measuring the parameters under two conditions of field distribution [6], [9].

Thus this proposed new method which enables measurement of ϵ' and tan δ over a wide range of values can be used with certain attachment for the whole range of these parameter's values. When the size of the sample is conveniently limited the proposed method proves to be superior to the earlier methods. Convenience of thermal shielding of the experimental sample makes it possible to study the dependence of ϵ' and tan δ with respect to temperature, which has important scientific significance regarding investigation of the properties of materials.

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A Spectral Domain Analysis for Solving Microstrip Discontinuity Problems

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Abstract—**A general approach for deriving quasi-static equivalent circuits for discontinuities in microstrip lines is presented. The formulation is based upon Galerkin's method applied in the Fourier transform domain. Numerical results are presented for a number of different configurations and compared with data available from other sources.**

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I. INTRODUCTION

THE PURPOSE of this paper is to develop a general approach for computing the discontinuity capacitance for a wide variety of microstrip structures. Currently, a number of different methods exist for attacking this problem, e.g., the moment method, the variational approach, projective method of solution for the integral equation, to list a few. Discussion of these methods may be found in publications by Farrar and Adams [1], Maeda [2], and Silvester and Benedek [3], [4]. The approach to be